Solution Set 5

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1 Problem 1

The irradiance for two interfearing waves is given by Eq. 3.3 to be

$$I(\delta) = |\vec{E}_1|^2 + |\vec{E}_2|^2 + 2\vec{E}_1 \cdot \vec{E}_2 \cos\delta.$$

In the case of the Fraunhofer diffraction the phase difference between two waves is $\delta = k\Delta x = kh\sin\theta = kh\sin(Y/D)$. (And we also assume that two waves have the same amplitudes $E_1 = E_2 = E$, in order to be able to express the irradiance as a function of just I_0 and δ).

a).

$$\vec{E}_1 \cdot \vec{E}_2 = 0$$

$$I(\delta) = E^2 + E^2 = 2I_0$$

b). Now we have a polarizer in front of the screen and both of the waves come out parallel to it, but with reduced amplitudes $E \rightarrow E\cos\theta$, where θ is the angle the field makes with the polarizer. Thus $E_1, E_2 \rightarrow E/\sqrt{2}$, and

$$I(\delta) = E^2/2 + E^2/2 + E^2\cos\delta = I_0(1 + \cos\delta)$$

c). This is the same situation as b), except $E_1 \rightarrow E/\sqrt{2}$ and $E_2 \rightarrow -E/\sqrt{2}$. Thus

$$I(\delta) = E^2/2 + E^2/2 - E^2\cos\delta = I_0(1 - \cos\delta)$$

2 Problem 2

a). The order of interference is given by Eq. 4.10 to be

$$N = \frac{2nd}{\lambda_o}\cos\theta + \delta_r,$$

where δ_r is the phase change for the reflected wave (and it is 0 since the reflections are taking place inside of the thicker medium). Thus

$$N = 3.3 * 10^{-5}\cos\theta.$$

b). Equations 4.13-4.14 tell us that

$$T_{max} = \frac{T^2}{(1-R)^2}$$

$$T_{min} = \frac{T^2}{(1+R)^2},$$

where $R = |r|^2$. Thus the ration is

$$\frac{(1-R)^2}{(1+R^2)} = 0.0028.$$

c). Using Eq. 4.18 the resolving power is

$$\frac{\omega}{\delta\omega} = \frac{ckd\sqrt{R}}{c(1-R)} = \frac{2\pi d\sqrt{R}}{\lambda_o(1-R)} = 2.2 * 10^{-7},$$

where $\omega = ck$ was used.

3 Problem 3

Here we can look at the transmitted field (using a variation of Fig. 4.1) to get

$$E_T = E_0 t_1 t_2 \sum_n (r_1 r_2)^n = \frac{E_0 t_1 t_2}{1 - r_1 r_2} = 2E_0\sqrt{n}/(1+n),$$

where we used the facts that $|r| = \frac{n-1}{n+1}$, $T = 1 - R = t^2$ and if the algebra is done correctly the index of refraction of the film drops out. Then we can compare it to the case where the beam is incident on glass with no film and find that the transmittances in both cases are $\frac{4n}{(1+n)^2}$. (You can also use Eq. 4.32 with $kl = \pi$).

4 Problem 4

For this problem we use Eqns. 4.27-4.28 ($kl = \pi$ for both layers) and

$$M = M_1 M_2 = (\begin{matrix} -n_2/n_1 & 0 \\ 0 & -n_1/n_2 \end{matrix}). \tag{1}$$

As a result we get equations

$$1 + r = -n_2/n_1 t = -1.27t$$

and

$$1 - r = -n_T n_1/n_2 = -1.19t$$

the solution to which is $|r|^2 = 0.00095$.

5 Problem 5

With the use of $\phi_n = \phi_1 + (n-1)\Delta\phi$, and remembering that summing from 1 to N is equivalent to summing from 1 to infinity minus the sum from $N+1$ to infinity we have

$$\sum_{n=1}^{N} e^{i\phi_n} = e^{i\phi_0}\sum_{n=1}^{N}(e^{i\Delta\phi})^n = e^{i\phi_0}\left(\frac{e^{i\Delta\phi}}{1-e^{i\Delta\phi}} - \frac{(e^{i\Delta\phi})^{N+1}}{1-e^{i\Delta\phi}}\right)$$

Then using $e^{i\theta} - 1 = 2ie^{i\theta/2}\frac{e^{i\theta/2}-e^{-i\theta/2}}{2i} = 2ie^{i\theta/2}\sin\theta/2$ we have

$$\sum_{n=1}^{N} e^{i\phi_n} = e^{i\phi_1}\left(\frac{1-e^{iN\Delta\phi}}{1-e^{i\Delta\phi}}\right) = e^{i(\phi_1+N\Delta\phi/2-\Delta\phi/2)}\frac{\sin N\Delta\phi/2}{\sin\Delta\phi/2}.$$

Since $\bar{\phi} = (\phi_0 + (N+1)\Delta\phi/2)$, we have proven the result.

6 Problem 6

This situation is the same as Problem6 on HW4, except all of the slits have the same width. Repeating the analysis we have

$$\vec{E}_s = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = \vec{E}_0 e^{i(kz-wt)}(1 + e^{i\delta} + e^{2i\delta})$$

Thus

$$|\vec{E}_s|^2 = \vec{E}_s\cdot\vec{E}_s^{\,*} = \vec{E}_0\cdot\vec{E}_0^{\,*}(1+e^{i\delta}+e^{2i\delta})(1+e^{-i\delta}+e^{-2i\delta}) = |\vec{E}_0|^2(3+2e^{i\delta}+e^{2i\delta}+2e^{-i\delta}+e^{-2i\delta}) =$$

$$= |\vec{E}_0|^2(3 + 4\cos\delta + 2\cos 2\delta) = I(\delta),$$

where $\delta = kh\sin\theta = kh\theta$ is the phase difference between the neighboring slits. The maximum irradiance is at the center ($\delta = 0$), therefore

$$I(\delta) = \frac{I_{max}}{9}(3 + 4\cos\delta + 2\cos 2\delta)$$

a).

$$I(\pi) = \frac{I_{max}}{9}$$

b). The cosines average out to 0. Thus $I_{av} = \frac{I_{max}}{3}$ and $I(0)/I_{av} = 3$.

7 Problem 7

a). For a double slit the irradiance is given in Eq. 5.27 to be

$$I(\theta) = I_0\left(\frac{\sin\beta}{\beta}\right)^2\cos^2\gamma,$$

where $2\beta = ka \sin\theta$ and $2\gamma = kh \sin\theta$ (a is the slit width and h is the slit separation). The (interference) maxima occur at $\gamma = n\pi$, but the fourth one is missing because $\sin\beta = 0$ at its location. Thus

$$k \sin\theta = \frac{2\gamma}{h} = \frac{8\pi}{h} = \frac{2\beta}{a} = \frac{2\pi}{a}.$$

Thus $h = 4a = 0.4mm$.

b). The maxima occur at $\gamma = n\pi$ and therefore at $\beta = ak \sin\theta/2 = n\pi a/h = n\pi/4$. Then $I_1/I_0 = \frac{8}{\pi^2}$, $I_2/I_0 = \frac{4}{\pi^2}$, $I_3/I_0 = \frac{8}{9\pi^2}$.

8 Problem 8

From Eq. 5.24 we can see that for a circular aperture

$$I = I_0(2J_1(\rho)/\rho)^2,$$

where $\rho = kR \sin\theta = kRr/D$ (r is the distance from the center on the screen and D is the distance to the screen, i.e. the focal length). The radii of the first two rings correspond to the first two maxima of $J_1(\rho)$. Thus $r = \rho D/(kR) = 0.015mm, 0.025mm$, where the light was estimated to have a wavelengh of $500nm$.